Repetitive Control to the Flow Rate of Peristaltic Blood Pumps

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Abstract: Most peristaltic blood pumps in hemodialysis machines currently use open-loop control, which leads to a relatively high perturbation level of flow rate. For some reasons, the blood flow rate from the peristaltic pump can't be measured directly. In this research, two simple but effective methods including empirical formula and polynomial curve fitting are applied to describe the relationship between the blood flow rate and RPM of the pump head. Both of the two methods perform well in describing the relationship. But the modified empirical formula is preferred because of its relatively low computational complexity. Taking RPM of the pump head as an indirect feedback of the flow rate, a robust repetitive control method is applied to lower the fluctuation level of the blood flow rate. And the result exhibits a better performance when it is compared with the general repetitive control.

Keywords: Peristaltic pump, hemodialysis machine, blood flow rate, robust repetitive control.

I. INTRODUCTION

Peristaltic pumps are excel at transferring extremely viscous flows and non-Newtonian fluids [1]. Operating principle of them is quite simple, so is their construction shown in Fig. 1. A distensible tube is fitted between the internal circumference of the pump housing and several (typically two) rollers attached to a pump head [2]. As the pump head rotates, the rollers that squeezing the tube will propel the fluid to move forward [3].
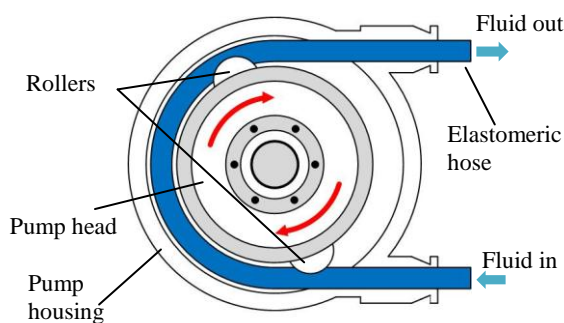

Fig. 1 Structure of a peristaltic pump

Minimize the pulsation peristaltic pumps generate is vital for patients [4], and the most common solution is getting feedback of flow rate at the outlet of the pumps by using flow meters. However, because the blood contamination is not allowed, it may be unrealistic to apply ordinary flow meters since they will inevitably contaminate the blood. Optical flow meters are probably the settlement, but they are always expensive.

Revolutions Per Minute (RPM) of the blood pump, which is directly related with the flow rate, is relatively convenient to be measured.

If we find a proper model to represent the relation, flow rate will be measured without contacting the blood stream. In this study, a bond between the two variables by trying several mathematical models will be formed. After the model is determined, a robust repetitive control algorithm will be applied to lower the perturbation level with RPM of the pump head.

II. METHODS

This section is mainly composed of three parts: the first is the devices and methods used for data acquisition, the next is the candidate mathematical models that are frequently utilized in curve fitting, and the last is the introduction of robust repetitive control.

A. Devices and Data Acquisition

A block diagram of our experimental set is shown in Fig. 2, which includes a load cell, two buckets, a peristaltic pump, NI USB-6008, PC and other necessary peripheral units. The peristaltic pump is driven by a 24 DC motor from FRANCK Inc., which has a build-in tachometer. Its pump head originally has three rollers.

The embedded tachometer send the data to NI USB-6008, and the data is further transmitted to PC through USB. Bucket 2 is put on a load cell that is connected to the AD converter HX711. And HX711 communicates with PC with the assistant of MCU. Flow rate of the peristaltic pump is given by differential of weight of Bucket 2 in respect to time. Programming language applied here is LabVIEW 2013. It sends out instructions to NI USB-6008, and the corresponding ports of the DAQ will inform the motor drive in what speed and direction the motor should actuate.
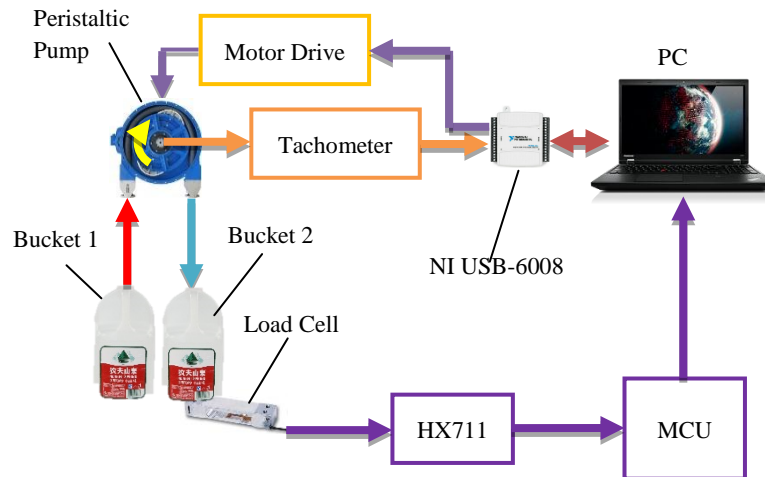
Fig. 2 Block diagram of the experimental set

B.   Data Processing

1)   Empirical Formula:
Flow rate of the peristaltic pump varies with many factors. Occlusion, the percentage that the tube is compressed, affects both pumping performance and lifespan of the tube. Its impact on flow rate takes place when the hose is not fully squeezed, especially under high pressure. In this case, the fluid being pumped is tend to remain still or even slip back. Thus, flow rate will be different even if RPM is the same.

Inside radius of the tube is also an important parameter. Apparently, larger inside radius means larger cross section area of the tube, and larger volume of fluid will be pumped during one period of rotation. The impact of outside radius of the pump head on the flow rate is also intuitive. As shown in Fig.1, longer section of the tube will be compressed as outside radius of the pump head increases. Last but not least, RPM of the pump head will directly affect how fast the fluid flows. Generally, more rollers will slightly decrease the flow rate, which can be ignored.

In conclusion, blood flow rate (BFR) [mL/min] of our peristaltic pump can be measured by the flowing equation:

$$BFR = RPM \cdot n \cdot \pi r^2 \cdot \frac{2\pi R}{n} \cdot 10^6 = RPM \cdot 2\pi^2 r^2 R \cdot 10^6 \quad (1)$$

Where r [m] is the inside radius of the tube, and R [m] is the outside radius of the pump head. The parameter "n" stands for n rollers, and "10^6" converts m^3/min to mL/min. Because the tube in our pump is fully squeezed, occlusion is not taken into consideration [5].

2)   Polynomial Curve Fitting:
Since there is always some disturbance, the linear function expressed as Eq. 1 may not describe the real relationship between RPM and the flow rate. In this case, polynomial curve fitting can be applied to construct a curve that has the best fit to a series of data points acquired by experiments.

Its main task is to find the proper parameters of the kth degree polynomial:

$$y = a_0 + a_1 x + \cdots + a_k x^k \quad (2)$$

Suppose y_i is the ith observed data, then the residual is given by [6]:

$$R^2 = \sum_{i=1}^{n} [y_i - (a_0 + a_1 x + \cdots + a_k x_i^k)]^2 \quad (3)$$

The partial derivatives are:

$$\frac{\partial R^2}{\partial a_0} = -2\sum_{i=1}^{n} [y - (a_0 + a_1 x_1 + \cdots + a_k x^k)] = 0 \quad (4)$$

$$\frac{\partial R^2}{\partial a_1} = -2\sum_{i=1}^{n} [y - (a_0 + a_1 x_1 + \cdots + a_k x^k)]x = 0 \quad (5)$$

$$\frac{\partial R^2}{\partial a_k} = -2\sum_{i=1}^{n} [y - (a_0 + a_1 x_1 + \cdots + a_k x^k)]x^k = 0 \quad (6)$$

These lead to the following equation:

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^k \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^k & \sum_{i=1}^{n} x_i^{k+1} & \cdots & \sum_{i=1}^{n} x_i^{2k} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^k y_i \end{bmatrix} \quad (7)$$

If n points (x_i, y_i) are observed, an equation set in the form of matrix can be obtained:

$$\begin{bmatrix} 1 & x_1 & \cdots & x_1^k \\ 1 & x_2 & \cdots & x_2^k \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (8)$$

Or:

$$xa = y \quad (9)$$

Premultiplying both sides with x^T, we can get:

$$x^T xa = x^T y \quad (10)$$

This is another form of Eq. 7. Finally, the least square solution of vector a can be solved by:

$$a = (x^T x)^{-1} x^T y \quad (11)$$

Selecting a proper degree of polynomial carefully makes sense, which serves the purpose of avoiding overfitting. Overfitting is the use of models or procedures that violate parsimony—that is, it include more terms than are necessary or use more complicated approaches than are necessary. Generally, the overfitting problems can be divided into two categories. Using a model that is too flexible than necessary. Or using a model that contains irrelevant components. Polynomials with excessive degrees will easily undergo the second type of overfitting problem. The best degree of polynomials is figured out by the trial-and-error method in this work.

3)   Robust Repetitive Control:
Repetitive control is first introduced by Inoue in 1981 [7]. It is particularly useful for suppressing disturbances with fixed period-time. When the setpoint follows a predetermined trajectory, repetitive control is often in the form of semi-open-loop system, which is known as iterative learning control. Sometimes periodic measurement deviation is obvious. In this circumstance, repetitive control is utilized as digital comb-filters [8].

The structure of a general closed-loop control system with a repetitive controller is shown in Fig. 3. Period-time of the reference signal *r* is constant and known. The block named "Memory Loop" is a delay line with length equal to the period-time of the external signals. A memory loop can be used to generate an output signal at frequencies $k\omega_p$ where *k* is an integer and ω_p is the period frequency. As is mentioned before, both the reference and external signals are required to have fixed and known period-time [9]. In peristaltic pumps, the period-time of external signals are always unknown and varies with time. Therefore, a new structure of repetitive control system is implemented in this work.
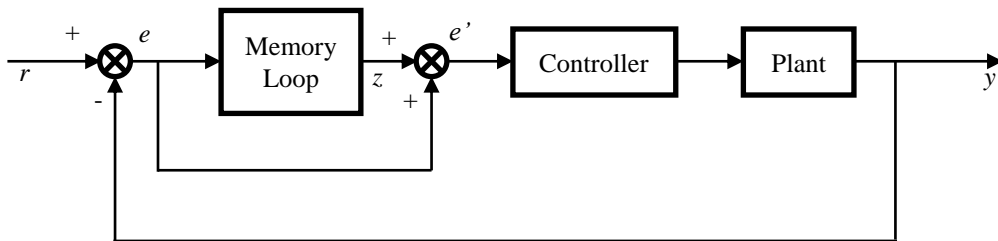

Fig. 3 Block diagram of the general repetitive control system

Fig. 1 shows the generalized repetitive controller that consists of three periodic signal generators. Weighting factors here, w_1, w_2 and w_3, are used for the purpose of modifying the dynamic response in between the harmonic frequencies. Assume the generalized repetitive controller has *N* periodic signal generators, then the loop transfer function *H(s)* can be written as:

$$H(s) = \sum_{i=1}^{N} w_i e^{-isT_p} \qquad (12)$$

Thus the transfer function of the controller can be written as:

$$G(s) = \frac{z}{e} = \frac{H(s)}{1 - H(s)} \qquad (13)$$

Basically, the working principle of the repetitive controller is to have infinite loop gain at the harmonics of the disturbance. Thus, *G(s)* is expected to increase to infinite at the harmonics. That is, in Eq. 13, $s=jk2\pi/T_p$. And we can infer from the above condition that the sum of all the weighting factors is 1. To avoid the limits of the standard repetitive controller, variations of T_p is expected to have no impact on the change of *H(s)* when $s=jk2\pi/T_p$:

$$\frac{\partial H(s)}{\partial T_p} = 0 \qquad (14)$$

In other words:

$$\frac{\partial H(s = jk2\pi/T_p)}{\partial T_p} = -jk2\pi/T_p \sum_{i=1}^{N} w_i i = 0 \qquad (15)$$

It can be implied from Eq. 15 that:
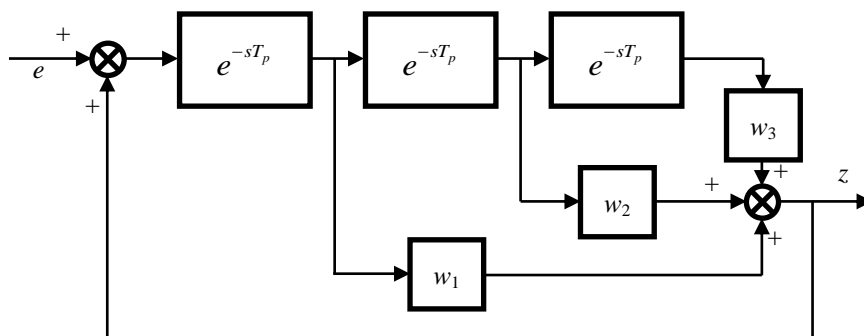
$$\sum_{i=1}^{N} w_i i = 0 \qquad (16)$$

Fig. 1 Memory-loop that contains three periodic signal generators

According to Fig. 1, *N* memory loops will have *N* corresponding weighting factors. To find all the unknown weighting factors, *N* constrains are required. Besides Eq. 16 and the sum of all the weighting factors, the following equation can also be regarded as constrains:

$$\sum_{i=1}^{N} w_i i^{(N-1)} = 0 \qquad (17)$$

Similar to Eq. 16, Eq. 17 is derived from the (*N*-1)th derivative of *H*(*s*) with respect to *T*$_p$.

Usually, two filters *Q*(*s*) and *L*(*s*) are designed to guarantee the stability of the controller. *L*(*s*) is the learning filter that compensates the transfer of the controller. And *Q*(*s*) is utilized to monitor the deviation between *L*(*s*) and the real system. For more details, refer to the literature [10][11].

III. RESULTS AND DISCUSSION

A. Tests of the empirical formula and polynomial

Inside radius of the tube in our work is 0.00361 m, and the outside radius of the pump head is 0.0699 m. In accordance with Eq. 1, the relationship between the BFR and RPM should be:

$$BFR = 8.991 \cdot RPM \qquad (18)$$

Substitute the values of RPM into Eq. 18, and the theoretical BFR can be obtained.

Fig. 2 shows the mismatch between the theoretical and practical BFRs. The theoretical BFRs are relatively larger than the practical BFRs, which attributes to two reasons. Firstly, the volume of the rollers are neglected by Eq. 1. Secondly, cross-section of the tube is not a strict circle. Instead, it is close to an ellipse due to squeeze and stretch from the rollers [12].

Despite the increasing gap between the two curves as the RPM rises, ratio between the two can be regarded as constant as Fig. 2 (b) shows. Mean value of the ratios, or calibration factor, is 0.82832. Multiply the correction coefficient with Eq. 18, the calibrated theoretical curve is obtained (Fig. 2 (c)). Fig. 2 (d) gives another example of the comparison between the practical flow rate and the calibrated theoretical BFR. Regardless of the disturbances during flow rate measurement, the calibrated curve fits quite well with the practical curve.
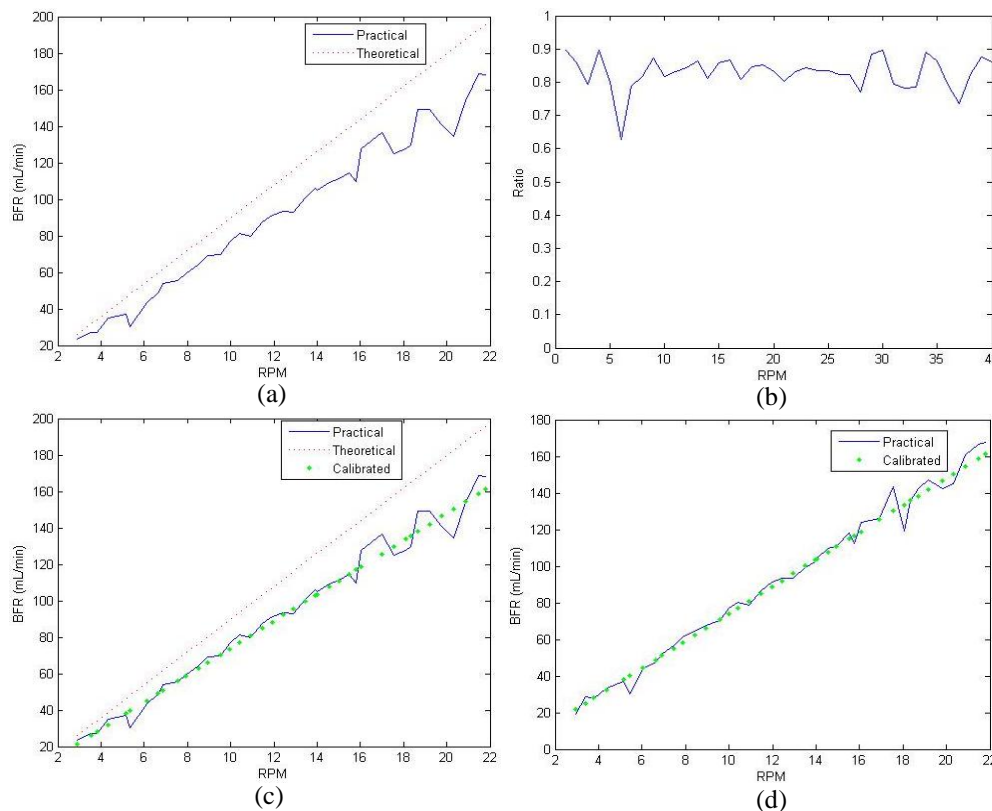

Fig. 2 Comparison between the practical and theoretical flow rate. (a) Before the theoretical curve is calibrated; (b) Ratio of the two under different RPMs; (c) After the theoretical curve is calibrated; (d) Another example.

Orders of polynomial applied here are also tested and compared with the empirical formula here. The result is shown in Fig. 3. We can see that the 1st degree polynomial whose parameters are determined by the principle of least square is almost the same with the empirical formula. Their MSEs are also close to each other with the former 22.9245 and the latter 24.3740. Rising the degree of the polynomial tend to reach a bit lower MSE, but it is not necessary because it will increase the complexity of calculation. For the same reason, the empirical formula, BFR=7.4474*RPM, is preferred. It only needs multiplication once while the polynomial, *y*=7.4918*x*-0.033, requires not only multiplication once but subtraction once.

B. Application of Repetitive Control to Blood Pumps

The application of repetitive control has been done with two memory loops (N=2). In this case, there are two unknown weighting factors, w_1 and w_2, left for us to find. According to Eq. 16 and $w_1+w_2=1$, the solution is easily to figure out: w_1=2, w_2=-1. The maximum rotating speed of the pump head is less than 25 rpm, which means that it is less than 1 Hz. Period-time of the disturbance is 0.156 Hz. Therefore, the sampling frequency is 10 Hz. FFT of the flow rate signal is shown in Fig. 4.
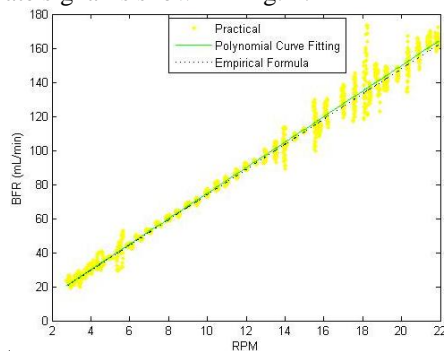

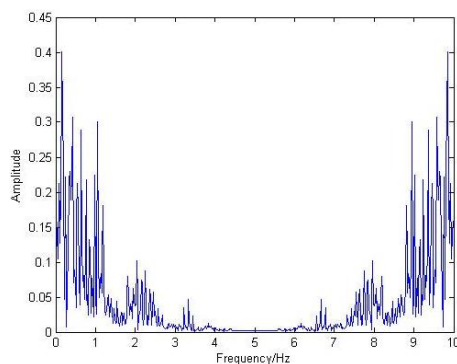Fig. 3 1st degree polynomial for fitting the practical points

Fig. 4 FFT of the original flow rate signal

It is clearly shown in Fig. 5 that the robust repetitive control method implemented here performs much better in suppression of perturbation when it is compared to the general repetitive control. The disadvantage of the robust repetitive control is the doubled storage need because of the extra memory-loop.

IV. CONCLUSION

Peristaltic blood pumps are vital components in hemodialysis machines. For some reasons, most blood pumps used in hemodialysis machines use open-loop control, which will lead to a relative high perturbation level. In this work, two methods including the empirical formula and polynomial curve fitting are firstly applied to describe the relationship between the flow rate and the RPM of the pump. Taking computational density and mean square error into consideration, the modified empirical formula is preferred. RPM of the peristaltic pump head is then regarded as the indirect feedback of blood flow rate. To lower the perturbation level while the period-time of the disturbance is not a constant, the robust repetitive control is implemented and the result shows that this method can effectively suppress the fluctuation of the flow rate.
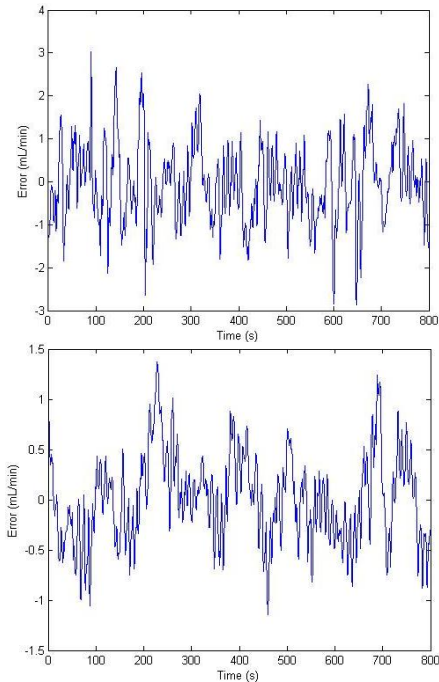

Fig. 5 Error signals of general repetitive control (top) and robust repetitive control (bottom) in time domain

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